UDC 537. 324 + 536. 23

A NOTE ON THE LIMITING THERMAL LOAD FOR FLASH TUBES

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Inzhenerno-Fizicheskii Zhurnal, Vol. 9, No. 1, pp. 105-111, 1965

An expression is derived for the dependence of the limiting discharge energy in flash tubes on the duration of the discharge. The influence of the physical properties of the wall material, the shape of the pulse, and the constructional features of the lamp is studied.

Although there are many possible causes for the failure of tubular gas-discharge flash lamps, all types of damage occur in a comparatively narrow range of flash energies [1-4]. Thus, the limiting load may be regarded as a quite definite quantity. Observation indicates that the main cause of overloading failure is damage to the wall, with the appearance of circular hair cracks on the inner surface of the tube. This is typical of thermal failure.

We shall examine a circular cylindrical tube, the inner surface of which is exposed to a nonsteady heat flux. We shall consider that the outer surface is thermally insulated, in view of the short duration of the process under consideration. The problem of finding the temperature $t(r, \tau)$ in an infinitely long tube amounts to integration of the equation of heat conduction

$$\frac{\partial t(r, \tau)}{\partial \tau} = a\left(\frac{\partial^2 t(r, \tau)}{\partial r^2} + \frac{\partial t(r, \tau)}{r\partial r}\right) \tag{1}$$

with initial and boundary conditions

$$t(r, 0) = t_i, \quad \left.\frac{\partial t(r, \tau)}{\partial r}\right|_{r=r_0} = -\frac{Q(\tau)}{\lambda}, \quad \left.\frac{\partial t(r, \tau)}{\partial r}\right|_{r=R} = 0. \tag{2}$$

The solution may be obtained by using a Laplace transformation. We write

$$\bar{t}(r, p) = \int_0^\infty t(r, \tau)\exp(-p\tau)\,d\tau.$$

Applying the Laplace transformation to (1), we obtain an equation for which the general integral, allowing for the initial condition, has the form [5-7]

$$\bar{t}(r, p) = \frac{t_i}{p} + c_1 I_0\left(\sqrt{\frac{p}{a}}r\right) + c_2 K_0\left(\sqrt{\frac{p}{a}}r\right). \tag{3}$$

Applying the Laplace transformation to the boundary conditions, we have

$$\left.\frac{\partial \bar{t}(r, p)}{\partial r}\right|_{r=r_0} = -\frac{1}{\lambda}\int_0^\infty Q(\tau)\exp(-p\tau)\,d\tau,$$
$$\left.\frac{\partial \bar{t}(r, p)}{\partial r}\right|_{r=R} = 0. \tag{4}$$

Putting (3) into (4), we obtain a system for determining constants c_1 and c_2. Substituting c_1 and c_2 into (3), we find

$$\bar{t}(r, p) = \frac{t_i}{p} + \frac{\sqrt{a}}{\lambda\sqrt{p}}\left[K_1\left(\sqrt{\frac{p}{a}}R\right) I_0\left(\sqrt{\frac{p}{a}}r\right) + I_1\left(\sqrt{\frac{p}{a}}R\right) K_0\left(\sqrt{\frac{p}{a}}r\right)\right] \times \left[I_1\left(\sqrt{\frac{p}{a}}R\right) K_1\left(\sqrt{\frac{p}{a}}r_0\right) - I_1\left(\sqrt{\frac{p}{a}}r_0\right) K_1\left(\sqrt{\frac{p}{a}}R\right)\right]^{-1} \int_0^\infty Q(\tau)\exp(-p\tau)\,d\tau. \tag{5}$$

To obtain solutions suitable for small values of the Fourier number [5-7], we can substitute an expansion of modified Bessel functions into (5), using the asymptotic formulas

$$I_n(x) = \exp(x)/\sqrt{2\pi x}, \quad K_n(x) = \sqrt{\pi/2x}\exp(-x). \tag{6}$$

These formulas are valid for large values of x. Thus, with an error of ≈ 5%, in the expansion

$$I_n(x) = \frac{\exp(x)}{\sqrt{2\pi x}}\left(1 + \frac{1}{8x} + \frac{9}{128x^2} + \ldots\right),$$

where $x = \sqrt{p/a}\, r_0$, we can retain only the first term, if

$$\sqrt{p/a}\, r_0 \gg 2.5. \tag{7}$$

Substituting (6) into (5), we obtain

$$\bar{t}(r, p) = \frac{t_i}{p} + \frac{\sqrt{a}}{\lambda\sqrt{p}}\sqrt{\frac{r_0}{r}}\,\frac{\operatorname{ch}\sqrt{p/a}(R-r)}{\operatorname{sh}\sqrt{p/a}(R-r_0)}\int_0^\infty Q(\tau)\exp(-p\tau)/d\tau. \tag{8}$$

The ratio $\operatorname{ch}\sqrt{p/a}(R-r)/\operatorname{sh}\sqrt{p/a}(R-r_0)$ may be represented, for $r \approx r_0$, as $\operatorname{cth}\sqrt{p/a}(R-r_0)$, which is close to 1 when *

$$\sqrt{p/a}(R-r_0) \gtrsim 3. \tag{9}$$

Then from (8) we obtain

$$\bar{t}(r, p) = \frac{t_i}{p} + \frac{\sqrt{a}}{\lambda\sqrt{p}}\int_0^\infty Q(\tau)\exp(-p\tau)d\tau. \tag{10}$$

The above discussion relates to a tube with a thermally insulated outer surface. It can be shown for a constant outer surface temperature t_c

$$\bar{t}(r, p) = \frac{t_i}{p} + \frac{\sqrt{a}}{\lambda\sqrt{p}}\sqrt{\frac{r_0}{r}}\,\frac{\operatorname{sh}\sqrt{p/a}(R-r)}{\operatorname{ch}\sqrt{p/a}(R-r_0)}\left(\int_0^\infty Q(\tau)\exp(-p\tau)d\tau\right) + \frac{t_c - t_i}{p}\sqrt{\frac{R}{r}}\,\frac{\operatorname{ch}\sqrt{p/a}(r-r_0)}{\operatorname{ch}\sqrt{p/a}(R-r_0)}. \tag{11}$$

The multiplier $\operatorname{sh}\sqrt{p/a}(R-r)/\operatorname{ch}\sqrt{p/a}(R-r_0)$ is close to 1 for $r \approx r_0$ and when (9) is satisfied. Then for $t_c = t_i$ (11) coincides with (10).

As an example, we shall examine a heat flux whose intensity varies exponentially (approximating the conditions of energy dissipation for a discharge in a low-inductance circuit):

$$Q(\tau) = A\exp(-\beta\tau). \tag{12}$$

The maximum value of the heat flux power A per 1 cm^2 of tube outer surface may be found by putting

$$A = \gamma P_0/S_0. \tag{13}$$

Substituting (12) into (10), we find

$$\bar{t}(r, p) = t_i/p + A\sqrt{a}/\lambda(p+\beta)\sqrt{p}.$$

$\frac{\exp(-\beta\tau)}{i\sqrt{\beta}}\,erf(i\sqrt{\beta\tau})$ is the original of the transform $\frac{1}{(p+\beta)\sqrt{p}}$. Expanding $erf(i\sqrt{\beta\tau})$ in series and using only the first term of the expansion (which gives a satisfactory approximation for $\beta\tau \approx 1$), we obtain

$$t - t_i = \frac{2}{\sqrt{\pi}}\,\frac{A\sqrt{a}}{\lambda}\sqrt{\tau}\exp(-\beta\tau) = \frac{2}{\sqrt{\pi}}\,\frac{\sqrt{a}}{\lambda}\sqrt{\tau}\,Q(\tau). \tag{14}$$

*For estimate (9) we put $\tau = \tau_0$. In the region of transforms this corresponds to $1/p = \tau_0$. In this case, for a quartz tube $a = 7.5 \cdot 10^{-3}$ cm^2/sec when $R - r_0 = 0.1$ cm, $\tau_0 \leq 0.15$ sec.

As a second example, we consider a heat flux with power varying as

$$Q(\tau) = 2A\beta\tau \exp(1 - 2\beta\tau). \tag{15}$$

This power variation approximates the discharge conditions in a circuit with considerable inductance. Substituting (15) into (10), we have

$$\bar{t} = \frac{t_i}{p} + \frac{2{,}7A\sqrt{a}}{\lambda}\frac{2\beta}{\sqrt{p}(p+2\beta)^2}.$$

Converting from the transform to the original [8], for $\beta\tau \simeq 1$ we obtain the following approximate expression:

$$t - t_i = \frac{2}{1.5\pi}\frac{\sqrt{a}}{\lambda}\sqrt{\tau}\,Q(\tau). \tag{16}$$

For poor thermal conductors short-duration thermal shock is propagated only in the surface layers, in which high compressive stresses are created for relatively small tensile stresses in the body of the material. If the surface is heated practically instantaneously from t_i to t, then at the initial instant of time we get a surface compressive stress [9-12]:

$$\sigma = -\alpha E(t - t_i)/(1 - \gamma). \tag{17}$$

Destruction of the walls will follow when

$$|\sigma| = \sigma_d. \tag{18}$$

From (14), (17), and (18) it follows that

$$Q_d(\tau) = \frac{\sqrt{\pi}}{2}\frac{\sigma_d(1-\nu)\lambda}{\alpha E\sqrt{a\tau}}. \tag{19}$$

Using (13), we can find the limiting energy W_l for a flash of duration $\tau_0 = 1/\beta$:

$$W_l = \frac{S_0}{\gamma}\int_0^{\tau_0} Q_d(\tau)\,d\tau. \tag{20}$$

From (19) and (20) we obtain

$$W_l = \frac{\sqrt{\pi}}{\gamma}S_0\frac{\sigma_d(1-\nu)\lambda}{\alpha E\sqrt{a}}\sqrt{\tau_0}. \tag{21}$$

From (16), (17), (18), and (20), for the heat flux whose power varies according to (15) we obtain

$$W_l = 1{,}5\frac{\sqrt{\pi}}{\gamma}S_0\frac{\sigma_d(1-\nu)\lambda}{\alpha E\sqrt{a}}\sqrt{\tau_0}. \tag{22}$$

Introducing the numerical coefficient k, which takes into account the shape of the flash, we can combine (21) and (22):

$$W_l = k\frac{\sqrt{\pi}}{\gamma}S_0\frac{\sigma_d(1-\nu)\lambda}{\alpha E\sqrt{a}}\sqrt{\tau_0}. \tag{23}$$

The region of application of (23) is linked with the assumptions made in deriving it. The upper limit of flash duration is connected, in particular, with the satisfaction of (9) and (7); the lower limit depends on the fact that for flashes of duration 10^{-4} sec and less, especially in tubes of large diameter, one must take into account the stresses created in the walls by shock waves.

With the assumptions made in deriving (23), the limiting energy does not depend on the wall thickness. It was found in [1] that the dependence of the limiting energy on tube wall thickness fell within the limits of experimental error.

An expression for the limiting energy density W_l/V may easily be found from (23):

$$\frac{W_l}{V} = \frac{k}{\gamma}\frac{2\sqrt{\pi}}{r_0}\frac{\sigma_d(1-\nu)\lambda}{\alpha E\sqrt{a}}\sqrt{\tau_0}.$$

For quartz [13] $\alpha = 5.9 \cdot 10^{-7}$ degree^{-1}, $E = 7 \cdot 10^5$ kg/cm^2, $\nu = 0.16$, $\lambda = 0.017$ watt/degree $\cdot$ cm, $a = 7.5 \cdot 10^{-3}$ cm^2/sec. The destructive stress for quartz, allowing for the load being of short duration [14, 15], may be assumed to be twice that for static loading, i.e., $\sigma_d = 2000$ kg/cm². Then the formula for a quartz lamp takes the form

$$\frac{W_l}{V} = \frac{2.65 \cdot 10^3}{r_0} \frac{k}{\gamma} \sqrt{\tau_0}, \tag{24}$$

where k is a numerical coefficient, of the order of unity, depending on the shape of the flash.

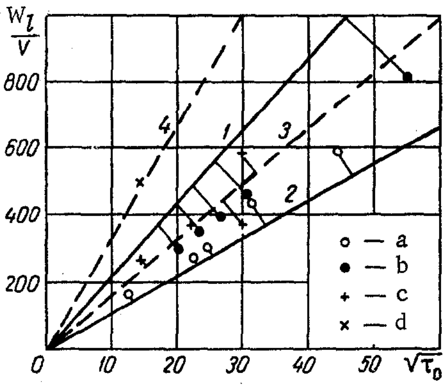

Fig. 1. Dependence of limiting energy density J/m³ on flash duration (μsec): a and b, c and d) experimental values from [1] and the present paper for a quartz lamp of internal diameter 0.9-1.2 cm and 0.4-0.5 cm (small L, $k \simeq 1$); 0.7-1.1 cm and 0.5 cm (large L, $k \simeq 1.5$); 1, 2 and 4, 3) according to (24) for lamps with internal diameter 0.5 cm, 1 cm, with $\gamma = 0.5$ cm, $k = 1$ and $\gamma = 0.5$ cm, $k = 1.5$.

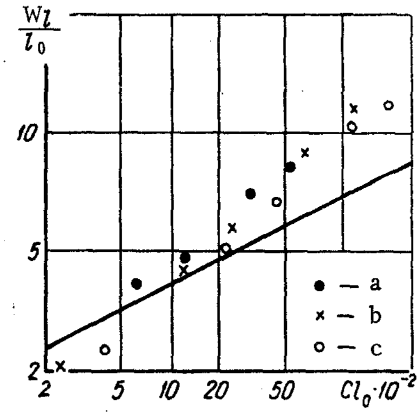

Fig. 3. Dependence of limiting energy per unit length of tube W_l/l_0 J/cm on the product of supply condenser capacitance and discharge tube length Cl_0, μF•cm: a, b, and c) experimental values from [1] for 3C-5 glass lamps with tube lengths of 15, 30, and 50 cm, and internal diameter of 0.4 cm. The solid line shows (25) for $\gamma = 0.5$ and $k = 1$. The relation between τ_0 and Cl_0 was found from [16].

The formula for a glass lamp ($\alpha = 4.9 \cdot 10^{-6}$ degree^{-1}, $E = 7 \cdot 10^5$ kg/cm², $\nu = 0.2$; $\lambda = 0.012$ watt/degree•cm, $a = 6.4 \cdot 10^{-3}$ cm²/sec, $\sigma_d = 2000$ kg/cm²) takes the form

$$W_l/l_0 = 760 r_0 \sqrt{\tau_0}\, k/\gamma. \tag{25}$$

Figures 1 and 2 show the theoretical dependence of limiting energy on flash duration for quartz lamps compared with the experimental data of references* [1, 4], and also with data obtained in the present study using equipment similar to that employed in [1, 4]. It can be seen from Figs. 1 and 2 that the theoretical dependence of limiting energy on flash duration is close to the experimental. A certain discrepancy between theory and the data of [1] for exponential flashes ($k \simeq 1$) at lamp diameters ≤ 0.5 cm may possible be connected with the assumptions made in deriving (23). A comparison of calculations with test data for glass lamps is shown in Fig. 3, where there is satisfactory agreement between theory and experiment. The relation between τ_0 and Cl used in constructing the graphs of Figs. 1 and 3 was found from [16].

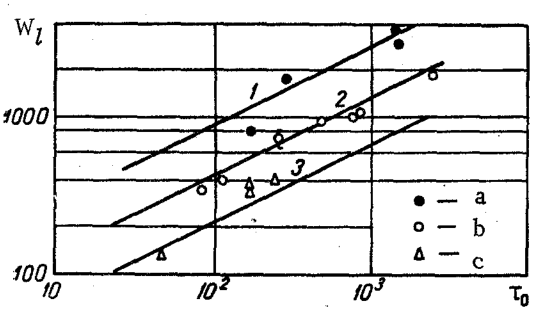

Fig. 2. Dependence of limiting flash energy on flash duration (τ_0, μsec): a, b, and c) experimental values from [4] for lamps with $l_0 = 15.2$, 7.6, and 3.8 cm and internal diameter 0.4 cm; 1, 2, 3) according to (24) for $\gamma = 0.5$ and $k = 1.5$, which corresponds approximately to the shape of the flash in [4].

We shall find the dependence of the destructive peak current J_0 on the flash duration τ_0.

We put

$$P_0 = J_0^2 R_c. \tag{26}$$

*In [1] and in our experiments, the dependence obtained was that of limiting energy on duration of light flash, which may be considered close to the duration of the heat flash. The dependence obtained in [4] was that of limiting energy on duration of current pulse. The light flash duration is close to that of the current pulse in operating regimes with large inductance $L > CR_c^2/4$.

From [3, 4], the resistance of the plasma column

$$R_c = \rho\, l_0/\pi r_0^2. \tag{27}$$

Replacing P_0 from (26) by the ratio W_l/τ_0, and taking account of (23), (26), and (27), we have

$$J_0 = \sqrt{2}\,\pi \left(\frac{\sqrt{\pi}\, k\, \sigma_p (1-\nu)\,\lambda}{\gamma \alpha E \rho \sqrt{a}}\right)^{\frac{1}{2}} \frac{r^{3/2}}{\tau_0^{1/4}}. \tag{28}$$

In Fig. 4 the dependence of J_0 on τ_0 from (28) is compared with the experimental data of [4]. There is satisfactory agreement between theory and experiment. The observed discrepancies are probably connected with some change in plasma resistivity with variation of flash length.

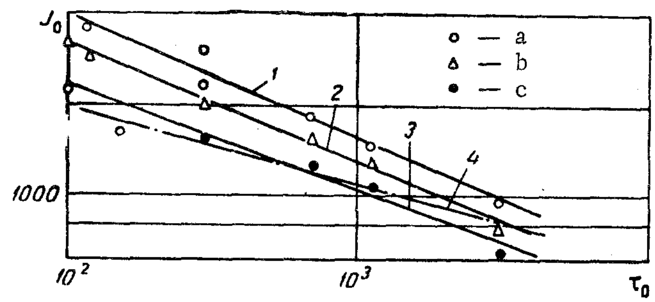

Fig. 4. Dependence of destructive peak current on the duration of the flash: a, b and c) experimental values from [4] for a tubular quartz lamp of length 7.6 cm and inside diameter 0.4 cm: 1) walls destroyed; 2) electrodes destroyed; 3) walls vaporized; 4) theoretical dependence according to (28) for γ = 0.5, k = 1.5 and ρ = 0.016 ohm • cm [4].

It follows from (28) that

$$J_0 \sim r_0^{3/2}.$$

This relation coincides with that found in [4]. We note that the dependence of W_l and J_0 on τ_0 given by (23) and (28) is approximate owing to the assumptions made in deriving these formulas.

The results of investigations of the limiting energy of glass and quartz lamps supplied by low-inductance circuits [1, 2] yielded the following empirical dependence:

$$CU_l^4/l l_0^3 = B, \tag{29}$$

where B is a constant.

From (29), taking account of (27) and bearing in mind the expression $\tau_0 \simeq R_c C/2$, valid for $\tau_0 \geqslant 500$ μsec [3, 16], it is easy to obtain

$$W_l = CU_l^2/2 = \sqrt{B/8\pi\rho}\, S_0 \sqrt{\tau_0}. \tag{30}$$

Comparison of (30) with (23) shows that the theoretical dependence of W_l on S_0 and τ_0 coincides with the empirical one, of which (29) is a generalization. In this case

$$B = 8\pi^2 k^2 \rho \sigma_d^2 (1-\nu)^2 \lambda^2/\gamma^2 (\alpha E)^2 a.$$

NOTATION

$t(r, \tau)$ — temperature at distance r from axis at time τ; r_0, R — inside and outside tube radii, respectively; $Q(\tau)$ — heat flux power per 1 cm² of inside surface of tube; t_i — initial temperature; λ, a — thermal conductivity and diffusivity, respectively; $\bar{t}(r, p)$ — transform of $t(r, \tau)$; $I_0(\sqrt{p/a}\, r)$, $K_0(\sqrt{p/a}\, r)$ — modified Bessel functions of zero order and first and second kind, respectively; τ_0 — flash duration ($\tau_0 = 1/\beta_0$); t_c — temperature of tube surface; A — maximum value of thermal flux power per 1 cm² of inside surface of tube; P_0 — peak flash power; S_0 — area of inside surface of tube; γ — fraction of flash energy converted into heat; σ_d — destructive stress; α — coefficient of linear expansion; E — modulus of elasticity; ν — Poisson's ratio; V — internal volume of tube; R_c — resistance of plasma column; ρ — resistivity of plasma; l_0 — tube length; C — capacitance of condenser supplying lamp; U_l — supply voltage corresponding to destruction of tube; L — inductance.

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